SOPHIE GERMAIN AND THE STRUGGLES OF WOMEN MATHEMATICIANS DURING THE FRENCH REVOLUTION

by

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Abstract

In this thesis, I analyze the social conditions of women during a time that they were not encouraged to become educated. Throughout the period of the French Revolution and the Enlightenment, women struggled to penetrate into scientific fields as they were denied access to a formal education. I specifically study the case of Sophie Germain and the perseverance that she exuded while acquiring a completely informal education. In conjunction with other scientists, Germain became one of the first known women of her time to make significant mathematical strides. She produced great work on elasticity and developed an innovative approach to Fermat's Last Theorem which led to a proof of the Sophie Germain Theorem. Sophie Germain, an early, innovative intellectual has left a lasting legacy in mathematics and has served as an inspiration for women who value education and desire to integrate into previously male-dominated fields.

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Contents

1 Introduction

In what follows we will briefly discuss the background of Sophie Germain. Then, we will review the French Revolution and the Enlightenment and analyze the rights of women during this time period. This analysis will then be used to examine the life of Sophie Germain in depth, showing her perseverance and motivation to become a highly educated individual that would contribute greatly to the field of mathematics. We will translate certain parts of the correspondences between Sophie Germain and Carl Friedrich Gauss, examining the rhetoric of their communication as it relates to the stigmas of the 18th and early 19th centuries. We will then delve into the mathematics of Sophie Germain which began with her work on Fermat's Last Theorem and has been extended and applied to other problems in number theory. We will conclude with the significance and impact that Sophie Germain has left on the field of mathematics.

2 Background on Sophie Germain

Figure 1: Illustration from 1880 of Sophie Germain at age 14.

An extraordinary female mathematician, Sophie Germain was born on April 1, 1776 in Paris, France. She died at age 55 on June 27, 1830, leaving behind one of the most significant mathematical contributions by any woman at this point in time. Germain, unlike her sisters, never married, not surprisingly, as the life she led was anything but conventional for her time [18]. Germain came from a financially stable family. Though her father was able to support her endeavors financially, she still suffered from her social standing. Germain did not fit into any group that could benefit her academically. She was not a male scientist, she was not welcomed among educated women as she was not of the upper class, and she was not related to a scientist who could consider her ideas; thus, Germain was forced to complete any education on her own. Sophie Germain worked mainly in number theory and the theory of elasticity. Some of her important contributions consist of identifying an important class of prime numbers, now called Sophie Germain primes, her work on Lagrange multipliers, and her new approach at proving Fermat's Last Theorem.

3 French Revolution

The French Revolution took place roughly between 1789 and 1799. The Revolution was prompted when Louis XVI dismissed his third estate representative [8]. The unrest that already existed among citizens who were not of the nobility and upper class was furthered, leading to protesting and rioting and eventually, the storming of the Bastille on July 14, 1789. The "deepening socioeconomic disparities and growing public resentment of the privileges enjoyed by the elite social and religious classes" fueled the fire towards the French Revolution [8]. Additionally, the Enlightenment brought about new political and social ideas, creating greater "opposition to the inequities of the feudal system" [8]. The French Revolution is officially documented as ending upon the coup and the coming to power of Napoleon Bonaparte; yet, the revolution of change itself continued much into the years of the Napoleonic Wars.

After the storming of the Bastille, the French government began working on a new constitution called the "Declaration of the Rights of Man and the Citizen." Through this document "the equality of civil rights for men and women was proclaimed" [11]. This document was intended to expand the rights of women, citizens, and slaves living in France, but many women were still unsatisfied, calling for a similar document to be titled the "Declaration of the Rights of Women and of the Female Citizen." Women felt as if they were still not a part of the debate and took it upon themselves, marching upon

Versailles, demanding “the principles of equality, justice and humanity which the Constitution owes me” [11]. In their proposed Declaration of the Rights of Women and of the Female Citizen, women desired their basic rights to be acknowledged, classified as the rights of liberty, property, safety, and especially resistance to oppression; these were “the natural and inalienable rights of woman and of man” [11]. Though this document was never officially ratified, women and men were beginning to change the way that they thought of women and their place in society.

The French Revolution was largely driven by underrepresented citizens of France. One French historian, Jules Michelet, described the Revolution as “so very spontaneous, unexpected, and truly working-class, mainly the affair of women” [11]. This spontaneity may have been part of the reason that progress was slow to come. Women and those of the working class knew that they desired change, but may not have been aware of how this change should take place. It was not until the Revolution of 1848 that the French achieved a constitutional monarchy. Women’s fight for change gradually increased the rights and opportunities of women until they became equal to those of men. The image below is kept today in the Louvre commemorating the Revolution of 1830 in which women continued to lead the French people to liberty.

Figure 2: *Liberty Leading the People* by Eugène Delacroix found at The Louvre in Paris.

4 Women's Rights During the Enlightenment and the French Revolution

Without delving past the surface, it would appear that education was valued highly during the eighteenth century as the French were in a period of Enlightenment and discovery. This may have been legitimate for men at the time, but it certainly was not the case for women. Germain was negatively affected by the Enlightenment views which inhibited her ability to access a formal education. Women were lacking in opportunity as a result of their social class, the strong presence of gender roles, and from the limitation of subjects that they were allowed to study. As a result, Gardiner mentions, "The role of women in science in eighteenth-century France is an almost totally unexplored field" [6].

Legally, there was compulsory education at the time, but as we have come to see now, this did not hold true for all people, nor did it serve as the proper means of opportunity and growth as it should have. "Parents were required to send their children to school until age fourteen...However, these decrees were never fully enforced, and illiteracy continued to be widespread, even among girls of the upper classes and the bourgeoisie" [16]. There were inequities between men and women and also among social classes. Spencer notes, "While the education of upper-class women is the most thoroughly documented, that of middle-class girls–few of whom became published authors–is less well known. As to the education of girls from poor families, it remains almost a total mystery" [16]. Furthermore, "the quality and scope of education varied widely from one region to another, with Paris ranking at the top of the literacy scale" [16].

From the available documentation, it is evident that upper class girls had the greatest opportunity among other women to learn. Despite this unique opportunity, society still had ways of deeming it unnecessary for upper class women to become educated. In Olympe's biography, Sophie Mousset shares an example, "It was no doubt assumed that her father's blood and fine mind would be a sufficient substitute for an education...It was also thought that her beauty would take care of the rest" [11]. Furthermore, Mousset says, "A woman only had to be beautiful or lovable; when she possessed these two advantages, she saw a hundred fortunes at her feet" [11]. There was consistently an excuse for why women did not need to be educated. It was commonly thought during the Enlightenment that the primary purpose of a woman was to please a man. It was thought that if she was beautiful, she had contributed appropriately to society.

Ultimately, any education that a woman did receive during the eighteenth century was a means of benefiting the male and the image of the family, rather than encouraging women to think intellectually and contribute to the academic society. Even certain females at this time had a distorted understanding of the importance of education. Mousset discusses Madame Roland's thoughts about "how the education of women could contribute to the betterment of men. She strongly supported the education of women, but in a disarming manner, as her ultimate goal was to make men happier, and therefore better" [11]. During the Enlightenment, the logic followed from the idea that "It is appropriate [that girls] have a minor acquaintance [with history] so that girls might be no more ignorant than ordinary people" [16]. Men wanted women that were sophisticated and could contribute to intellectual discussions among other distinguished individuals as not to embarrass their husbands by their lack of knowledge. Mousset says, "Female education must be in direct relation to men: to please them, to be useful to them, to be loved and honored by them...and make their life pleasant and gentle: those have been women's duties throughout time, and should be taught from childhood" [11].

The subjects that women were allowed to study during the Enlightenment and French Revolution were limited to the sphere which women were stereotyped into.

> "The most portentous economic and social changes of the eighteenth century directly marked the working lives of women. For the vast majority of women, their membership in a family economy and the prescribed roles of the dominant gender system continued to govern the work to which they would have access or be obliged to perform" [6].

Unfortunately, even the support for female education coming from a man's perspective did not guarantee their support in every subject.

> "Fénelon admitted that 'female stupidity stems from the lack of education from which women suffer,' which however did not keep him from recommending the restrictive control of female education, in order for women to remain focused on accomplishing their allocated tasks, household chores, and the like" [11].

Gardiner brings light to the truth that gender roles largely influenced what subjects were considered important for women to study. "Safe subjects at the time were considered reading, writing, basic arithmetic, crafts such as needlework and sewing, theater, and religion [16]. It was appropriate during the eighteenth century for girls to be sent to religious convents for education, so that they would receive an extremely focused education with religion at the forefront. At the convents, girls were able to study some of these safe subjects, useful home economical skills, and religious teachings. It is also explained that,

> "'The school master is the adjunct of the parish priest. His major responsibility is to train christians...What matters most is not his knowledge...but his regular attendance at church functions and the religious teachings...he is capable of giving.' Education of girls of all social classes was then almost exclusively secured by religious orders" [16].

Religion played a crucial role in society at this time; it was taboo not to devote oneself to the church which is reflected by the importance that was put on a religious education. Though women did not have many chances to dabble in science and other sophisticated fields, religion was considered "safe" and, therefore, acceptable for women to be fully educated in.

It was generally uncommon for women to be educated in any other realm outside of the convent for much of the eighteenth century. Even if a particular family had the resources to educate their daughters, it was frowned upon to do so. One father of a daughter born in 1748 felt, “He couldn’t have kept her at home and given her private tutors, as that would have provoked quite a scandal, especially in the largely Protestant town of Montauban” [11]. As women grew older, they had the opportunity to attend salons on their own accord. Mousset explains the positive impact that salons had for women saying,

> “Salons were very fashionable at the time...the précieuses [hostesses] took great interest in science...Politics, literature, science, philosophy, as well as history were discussed; great minds met and confronted one another, always respectful...these women allowed for a productive social mix and communication between the various social classes” [11].

Salons provided a space for women to discuss topics of their choice among other elite women and many men. Women were the hostesses of salons, giving them a leadership role in intellectual discussions which they would not have had otherwise. Salons marked a shift in the social status of women by allowing them to contribute to scientific thoughts of the time. Women who participated in salons were often bold, as they criticized privilege and discussed progress and reason after studying the work of Enlightenment philosophers, and they stood firm and were strong-willed in their beliefs [11].

As the 1700s progressed, women began to desire a voice in their future more and more, recognizing the importance of education and its ability to bring about a complete understanding of the world. Women were noticing the severe need for reform in the education sector for women. They began to express this through passionate writings. “Their writings, in most cases, were aimed at female readership and were intended to fill the gaps of formal education and to contribute to the overall intellectual development of women” [16]. Even the smartest women were not satisfied with the education they had received and suggested new methods and more appropriate programs.

One popular method of publicly expressing contradictory ideas was through plays or stories, as the theater was a popular place within society and this method allowed authors to hide their identity behind the characters within the piece. For example, one included line in a play was, “I feel that the qualities that are habitually assigned to both sexes are not set in a fair manner” [11]. In one story by Olympe, it was stated, “The same education should be offered to young ladies as to young gentlemen” [11]. These authors made an effort to express their radical opinions without ridicule by using literature, but not all authors were successful in avoiding negative feedback.

In 1787, Marquis de Condorcet wrote a published letter vying for equal rights for women and men. Later in 1789 he wrote, “legislators have violated the principal of equal rights by quietly depriving half of mankind of its right to participate in the elaboration of laws” [11]. Condorcet was forced to withdraw his statement. More specifically, in 1758, Claude Adrien Helvetius wrote anonymously about the idea “that the equality between man and woman depended on the similarity of their brains; having the same organs to think with, they should therefore both be capable of acceding to culture...and that the most fundamental principal of equality resided in education” [11]. Like Condorcet, Helvetius’ statement was withdrawn by means of his book being banned and burned only a year later. The list of men supporting women’s rights was short and few; others fell on the complete opposite end of the spectrum. “The doctor Alfred Guillois carried out a medical and psychological study of women of the Revolution and believed that they were all ill” [11]. Women that spoke out in the early stages of the Revolution were not taken seriously, but even worse, men were forced to take back any opinions that may have supported the rights of women.

Mousset cites Choderlos de Laclos’ work, *The Education of Women*, which encourages women to take control and bring about change to their current access to education. He is very upfront stating that men do not have the will to change the lack of female rights as they are the creators of this injustice. He compares their situation to slavery, urging them to have courage during the Revolution to declare change [11]. Eventually, “women acquired an unspoken right to knowledge and education” [16]. Women began to feel free to openly express interest in fields that were outside of the “safe” subjects which they were previously limited to. Yet, despite this minor improvement, progress was ultimately slow to come. “Women were to wait three-quarters of a century after the Revolution before being allowed to complete their high school education” [16].

5 Sophie Germain’s Education

Sophie Germain was likely the first female mathematician of the time to successfully self-educate herself “at least to the undergraduate level” [10]. Female mathematicians prior to Germain had private tutors, but Germain did not have the same luxuries as these women. She spent many cold nights studying the mathematical books within her father’s library to teach herself despite her parents’ effort to deter her from her scientific interests. Germain would stay up long after her parents went to bed, studying by candlelight. One depiction of Germain's life shows "concerned parents depriving their daughter of candles and even heat, all in an effort to discourage her new interest" [18]. Yet, Germain persisted in her education, finding immense joy when she became confident in her understanding of new mathematical concepts. As her parents did not understand her interest in science, she, also, could not grasp their obsession with money and politics.

Germain's father was elected to the Estates General in 1789, resulting in a great deal of political discussion within the Germain home. Germain ignored the "political drama of the revolution and the social expectations of her family," instead, delving further into the works of Isaac Newton, Leonhard Euler, and other renowned mathematicians [18]. There is evidence that her father valued education as he was a member of the educated bourgeoisie society, but it took him some time before he valued and supported the education of his daughter. Being a member of the bourgeoisie gave the Germain family a sense of status within society, but did not necessarily classify them as upper class. One of bourgeoisie status was a leader in an industry, owning a sector of production, and had a level of cultural or financial capital that was of some significance. Germain's father worked as a merchant and eventually as a director of the Bank of France [14]. Eventually, as her father's endeavor in politics came to a close, he began to support her both financially and by hosting many scholars in their home.

Not only did Germain face struggles from the lack of support from her family, she also suffered from the lack of support in society. There was a gap between amateur and professional scientists which continued to widen due to the inequities between social classes and gender in regards to education. Spencer notes, "Women had no choice but to approach science as amateurs" [6]. Germain could hardly be considered even an amateur at this time as she had received no formal education, nor was she allowed to. It was not until 1794 that Germain had any documented contact with the public scientific sphere [10]. "To be active in science, then as now, required two things in particular: direct access to the people and institutions engaged in significant research and the financial resources for the acquisition of books and laboratory equipment" [6]. Her work in these areas stemmed from her initial contacts, mostly Joseph Louis Lagrange and Adrien-Marie Legendre, two prominent mathematicians at the time. As a female, Germain was unable to enroll in higher education; therefore, she took advantage of every opportunity she was given to obtain notes and knowledge. When presented with opportunities to work with male scientists, she was forced to study what they studied and worked on their problems. She was limited to the expertise of those willing to correspond with her, because she could not simply enroll in any university course that peaked her interest.

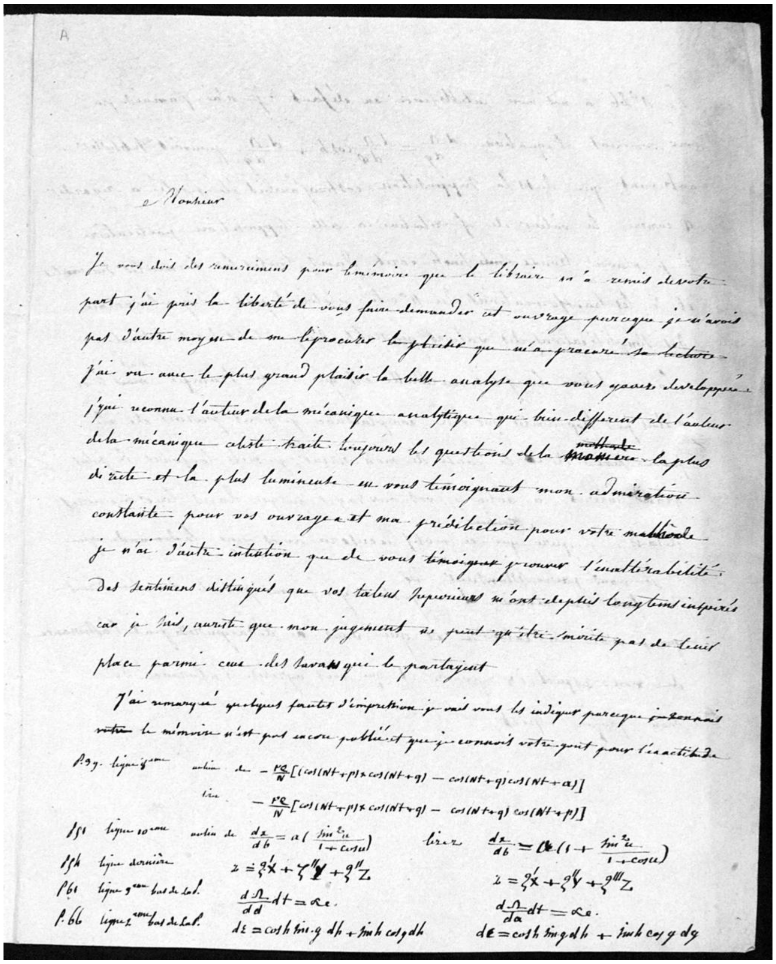
Even though women had slightly more freedom at this time, intellectual barriers still existed regarding access to normal institutions. “Women were never admitted to the presigious Académie...Their scientific contacts, then, had to be purely on the personal level, dependent on their noninstitutional relationships with practicing male scientists whom they met– again, largely accidentally– and whom they could persuade of their serious interest in the subject” [6]. An additional method that Sophie Germain utilized was the newly available public access to the notes of professors from the École Polytechnique. Students of the École Polytechnique could now offer their personal observations and thoughts on these notes. In an attempt to give her work legitimate consideration, Germain submitted her work as a male student, Monsieur Antoine August LeBlanc, after studying the analysis notes of Josepth Louis Lagrange. It did not take long for Lagrange to discover Germain’s secret identity, but her tactic of pretending to be a male proved successful for her as Lagrange continued his correspondence and work with her. She found herself quite lucky to stumble upon this opportunity, resulting in her first professional contact. Germain also initiated correspondences with Adrien-Marie Legendre and Carl Freidrich Gauss, a German mathematician. These two connections played crucial roles in Germain’s major contribution to mathematics. She studied number theory under Legendre and Gauss, eventually making progress on Fermat’s Last Theorem. Germain laid the foundation for one of the first general theorems for many cases, pertaining to what are known today as “Sophie Germain primes” [18]. Furthermore, her friendship with the mathematician, Joseph Fourier, led her to be “the first woman who was not a wife of a member to attend sessions of the French Academy of Sciences” [18].

Despite these new and resourceful contacts, Germain was not free from all pressures and inequalities at this time. “For example, when she was preparing her theory of elasticity and had to visit the École Polytechnique or Institute of France, she needed formal invitations and escorts. When she submitted a paper to the Academy of Sciences extending her theory in 1825, her submission was simply ignored” [18]. She was not always credited properly for the extent of her work, rather the mathematician whom she worked under received the main acknowledgements; each of these mathematicians showed respect for Sophie Germain and her work as they corresponded with her on a variety of significant mathematical subjects.

Later, Sophie Germain studied the philosophy of science and applied mathematics. In 1811, Germain submitted the only entry in a two-year prize competition for a mathematical theory of elasticity. “The institute acknowledged that her ‘experiments presented ingenious results’ but judged the rigor of her analysis inadequate and renewed the competition”[18]. Germain, again in 1813, was the only one to submit an entry in the renewed competition. This time, she was granted honorable mention for her work, but was not officially awarded the prize until after a second revision. Shortly before Germain died, Gauss recommended her for an honorary degree from the University of Göttingen, but in good fashion with the other struggles that she faced as a woman, the request was denied [18].

6 Translations and Analysis of Sophie Germain’s Communication with Other Scientists

Sophie Germain benefited greatly from her communication with Joseph Louis Lagrange, Carl Freidrich Gauss, Adrien-Marie Legendre, and Joseph Fourier. Adrien-Marie Legendre played a significant role in developing the Sophie Germain Theorem further towards publication. Though he did not agree with all of her work, Joseph Fourier, a friend of Germain, held a position at the Academy allowing Germain to integrate into the prestigious Parisian scientific society [2]. The École Centrale des Travaux Politiques (now École Polytechnique) opened in 1794, but denied access to women. Sophie Germain, motivated as she was, gained access to the lecture notes of Joseph Louis Lagrange as she found his work in analysis intriguing. Out of fear of disrespect towards her ideas, she submitted some of her personal observations in regards to his work in the disguise of a man, Monsieur Adrienne August LeBlanc, who was an actual student enrolled at the École Polytechnique at the time.



Monsieur

Je vous dois des remerciments pour le mémoire que le libraire m'a remis de votre part j'ai pris la liberté de vous faire demander cet ouvrage parceque je n'avois pas d'autre moyen de me le procurer ~~le plaisir que m'a procuré sa lecture~~ j'ai vu avec le plus grand plaisir la belle analyse que vous y avez développée j'y ai reconnu l'auteur de la mécanique analytique qui bien different de l'auteur de la mécanique celeste traite toujours les questions de la ~~maniere~~ méthode la plus directe et la plus lumineuse en vous témoignant mon admiration constante pour vos ouvrages et ma prédilection pour votre méthode je n'ai d'autre intention que de vous ~~témoigner~~ prouver l'inalterabilité des sentiments distingués que vos talens superieurs m'ont depuis longtems inspirés car je suis assurée que mon jugement ne peut qu'être ignoré pas de leur place parmi ceux des savans qui le partagent

J'ai remarqué quelques fautes d'impression je vais vous les indiquer parceque ~~je connais votre~~ le mémoire n'est pas encore publié et que je connois votre gout pour l'exactitude

P. 39. ligne 9eme au lieu de $-\frac{rc}{N}[(\cos(Nt+p)\times\cos(Nt+q) - \cos(Nt+q)\cos(Nt+\alpha)]$ lisez $-\frac{rc}{N}[\cos(Nt+p)\times\cos(Nt+q) - \cos(Nt+q)\cos(Nt+p)]$

P. 51 ligne 10eme au lieu de $\frac{dx}{db} = a(\frac{\sin^2 u}{1+\cos u})$ lisez $\frac{dx}{db} = a(1+\frac{\sin^2 u}{1+\cos u})$

P. 54 ligne derniere $z = \zeta' X + \zeta'' Y + \zeta'' Z$ $z = \zeta' X + \zeta'' Y + \zeta''' Z$

P. 61 ligne 9eme bas de la p. $\frac{d\Omega}{dd}dt = \alpha e.$ $\frac{d\Omega}{da}dt = \alpha e.$

P. 66 ligne 2eme bas de la p. $d\varepsilon = \cos h \sin g\, dh + \sin h \cos g\, dh$ $d\varepsilon = \cos h \sin g\, dh + \sin h \cos g\, dg$

Figure 3: The beginning of one of her letters to Lagrange.

Lagrange praised the ideas of Germain and was pleasantly surprised at the revelation of her gender. He and other scientists began to visit her home, as they were intrigued by a woman as intellectual as she. Many expressed positive thoughts towards her and encouraged her in various ways to continue her studies, though not all received her intellect well. Joseph Lalande was rather demeaning upon visiting Germain. He encouraged her to read *Astronomie des dames*, a study of astronomy intended for women which at the time meant a less technical version without many important details as society did not deem it necessary for women to be experts in any scientific field [4].

Germain's written communication with Gauss was another important method with which she supplemented her lack of a formal education. There exist 14 total letters between Gauss and Germain, 10 of which were written by Germain, and only four by Gauss [4]. Germain wrote her first three letters again under the name LeBlanc. Despite being well-received by some scholars, she still did not have complete confidence to write to him under her real name. In her letters, Germain discussed some of Gauss' work and praised him greatly. She also wrote about some of her theorems, asking for his advice on them. In Gauss' first two responses, he was polite, yet focused mostly on his work. One can see his politeness as he begins the letter saying, "Monsieur, il me faut vous demander mille fois pardon d'avoir laissé six mois sans réponse l'obligeante lettre dont vous m'avez honoré." (Sir, I must ask you a thousand times to excuse me for having let six months go by without response to the kind letter which you have honored me.) He recognizes his delayed response, making it known that he did not have any bad or rude intentions. He also closes his letter with kind regards, "Agréez, Monsieur, l'expression de ma haute considération." (Accept, Sir, the expression of my highest consideration[4].)

Gauss shows respect for Germain's work by saying, "J'ai lu avec plaisir les choses que vous m'avez bien voulez [sic] communiquer." (I have read with pleasure the things that you have wanted to communicate to me.) Yet, he makes no effort to expand or give feedback on her ideas. Instead, he is quite eager to discuss his own work, saying, "Certainement je me serais empressé de vous témoigner tout de suite combien m'est cher l'intérêt que vous prenez aux recherches auxquelles j'ai dévoué la plus belle partie de ma jeunesse." (Certainly, I would be eager to testify to you right now how dear the interest that you take in my research is to me, which I devoted the most beautiful part of my youth.) After only one introduction through a letter, he does not hesitate to ask Germain, or LeBlanc to his knowledge, for assistance in regards to finding the man whom Gauss sent copies of his work worth over 600 francs, without receiving a penny or even a letter in return. Gauss asks, "Peut-être vous pourriez me donner des renseignements par quel moyen, on pourrait engager cet homme à faire son devoir." (Maybe you could give me some information by which means, we could engage this man to do his duty [4].)

In his second letter, Gauss is still polite thanking Germain, or LeBlanc, for her letter. "Je profite de la complaisance de M. Grégoire pour vous offrir, avec beaucoup de remerciements pour toutes les communications de votre dernière lettre, un exemplaire d'un petit mémoire que j'ai publié en 1799 et qui probablement vous sera encore inconnu." (I benefit from the kindness of Mr. Grégoire to offer you, with many thanks for all the communications of your last letter, a copy of a small memoire that I published in 1799 and that you probably will still not know.) His response was very short, only two paragraphs, and again, he did not respond to any of her mathematics, which she explained in detail. Rather, he only briefly mentioned what he was currently working on, "Je suis à présent occupé à perfectionner quelques méthodes nouvelles par rapport aux calculs des perturbations planétaires." (I am currently busy perfecting several new methods relating to the calculations of planetary perturbations [4].)

In her fourth letter, Sophie Germain was forced to reveal her true identity to Gauss. The Napoleonic Army had occupied Brunswick, Germany, where Gauss lived, so Germain asked General Pernety, a commander of the French artillery and family friend, to ensure the safety and proper treatment of Gauss. When General Pernety went to the home of Gauss and informed him of who was protecting him, Gauss replied that he was unfamiliar with the name Sophie Germain [4]. In her next letter, Germain identified herself as his protector. For this Gauss was deeply appreciative, and the tone of his letter changed immensely. He became much more affectionate towards her, in one letter saying, "En vous remerciant de tout mon cœur pour votre derniére lettre et les intéressantes communications que vous m'y faites." (Thanking you with all my heart for your last letter and the interesting communications you share with me [4].)

He also commented in depth on her mathematics for the first time, giving them credit and truly considering them. He was impressed with her knowledge, expressing, "Les notes savantes, dont toutes Vos lettres sont si richement remplies, m'ont donné mille plaisirs. Je les ai étudiées avec attention, et j'admire la facilité avec laquelle Vous avez pénétré toutes les branches de l'Arithmétique, et la sagacité avec laquelle Vous les avez su généraliser et perfection [sic]." (The scholarly notes, that so richly fill your letters, gave me a thousand pleasures. I carefully studied them, and I admire the simplicity with which you have penetrated all branches of arithmetic and the sagacity with which you generalized and perfected them [4].)

He further showed the admiration he held for her as a woman. He began his letter following the revelation of her identity with:

> "Le goût pour les sciences abstraites en général et surtout pour les mystéres des nombres est fort rare: on ne s'en étonne pas; les charmes enchanteurs de cette sublime science ne se décèlent dans toute leur beauté qu'à ceux qui ont le courage de l'approfondir. Mais lorsqu'une personne de ce sexe, qui, par nos mœurs et par nos préjugés, doit rencontrer infiniment plus d'obstacles et de difficultés, que les hommes, à se familiariser avec ces recherches épineuses, sait néanmoins franchir ces entraves et pénétrer ce qu'elles ont de plus caché, il faut sans doute, qu'elle ait le plus noble courage, des talents tout à fait ex-

traordinaires, le génie supérieur." (The taste for abstract sciences in general and mostly for the mysteries of numbers is very rare: this is no surprise; the enchanting charms of this sublime science are revealed in all their beauty only to those who have the courage to deepen it. When a person of this sex, who, because of our traditions and prejudices, must encounter infinitely more obstacles and difficulties than men, to familiarize oneself with this thorny research, nevertheless overcomes these impediments and penetrates the most hidden in them, she undoubted shows evidence of the most noble courage, extraordinary talents, superior genius [4].)

Gauss was so pleased with the knowledge of Sophie Germain and her willingness to protect him during wartime that he suddenly felt a true friendship between them and even noted himself as her "sincerest admirer." He asked of her in both of his final two letters to remain friends, saying, "Continuez, Mademoiselle, de me favoriser de Votre amitié et de Votre correspondance, qui font mon orgueil, et soíes [soyes] persuadée, que je suis et serai toujours avec la plus haute estime. Votre plus sincère admirateur Ch. Fr. Gauss." (Continue, Miss, to honor me with your friendship and correspondence, and be assured that I have and will always have the highest esteem for you. Your sincerest admirer, Ch. Fr. Gauss.) He also remarked, "Continuez de temps en temps de me renouveler la douce assurance que je puis me compter parmi le nombre de vos amis, titre duquel je serai toujours orgueilleux." (Continue from time to time to renew in me the kind assurance that I can count myself among the number of your friends, title of which I will always be proud [4].)

7 Fermat's Last Theorem

Pierre de Fermat made great contributions to the field of mathematics, mainly in number theory. His work includes the method of infinite descent, results on divisibility, sums of squares, Diophantine equations, and the first derivative test for maxima/minima in calculus [9]. In 1637, in Fermat's copy of Bachet's translation, he claimed, "It is impossible to separate a cube into two cubes, or a biquadrate into two biquadrates, or in general any power higher than the second into powers of like degree [13]," and then he noted noted that the margins were "unfortunately too narrow" to include proofs [9].

Fermat's Last Theorem. The equation

$$\boxed{X^n + Y^n = Z^n} \tag{1}$$

has no positive integer solutions for any natural number $n > 2$.

This mathematical equation is known as a Diophantine equation, because it only considers integer solutions. Fermat based his equation off of the case $n = 2$ which is well known to many of us as the equation that appears in the Pythagorean Theorem [1]. For $n = 2$ there exists many integer values x, y, z that are solutions to the equation $x^2 + y^2 = z^2$ and are called Pythagorean triples [9]. One of the most well known set of Pythagorean triples are 3, 4, 5 since $3^2 + 4^2 = 9 + 16 = 25 = 5^2$. A more complex and impressive set of Pythagorean triples are 168, 224, 280 since $168^2 + 224^2 = 28,224 + 50,176 = 78,400 = 280^2$.

The early attempts at Fermat's Last Theorem considered proving the equation had no positive solutions for specific values of n. Leonhard Euler used the method of infinite decent to prove Fermat's Last Theorem for $n = 3$. Fermat, himself, utilized known relationships of Pythagorean triples in order to prove Fermat's Last Theorem is true if $n = 4$ [9]. Other mathematicians of the time tried to prove Fermat's Last Theorem for other exponents using Euler's and Fermat's methods. To prove Fermat's Last Theorem, it was known to be sufficient for one to prove that this is true for $n = 4$ and every odd prime integer. Other powers can be eliminated by simple factoring. In order to see that we only need to prove Fermat's Last Theorem for every odd prime integer we will observe that a solution to Fermat's Last Theorem for a specific exponent can generate a solution for a factor of the exponent. Let us first consider multiples of 3 for Fermat's Last Theorem.

Theorem 1. If equation (1) has a solution for $n = 3k$, then it has a solution for $n = 3$.

Proof. Assume (x, y, z) is a solution for the equation $x^n + y^n = z^n$ for $n = 3k$. So, $x^{3(k)} + y^{3(k)} = z^{3(k)}$. This implies $(x^k)^3 + (y^k)^3 = (z^k)^3$, so (x^k, y^k, z^k) is a solution of positive integers for exponent 3. □

We know that if Fermat's Last Theorem is true for $n = 3$, we know it is true for any multiple of

3, thus we will begin by proving Fermat's Last Theorem for 3 and all multiples of 3. This means that since Euler proved equation (1) has no solution for $n = 3$, it has no solutions for any multiple of 3. Likewise, a solution of (1) for any composite number will yield a solution for any factor. Take $n = 75$ for example, and see that $75 = 15 * 5$ so a solution for $n = 75$ would give a solution for $n = 15$ and $n = 5$. Thus, we can conclude:

Theorem 2. If n has prime factor k, then a solution for n leads to a solution for k.

Proof. Assume there is a solution for n, so that $x^n + y^n = z^n$. If $n = k * m$, and $k \geq 3$ and k is prime. Then, $(x^m)^k + (y^m)^k = (z^m)^k$, so there is a solution for k. If the theorem holds for n, then it holds for k. Likewise, if it fails for k, it fails for n. $\square$

This theorem shows if Fermat's Last Theorem is true for odd primes, then it is true for any multiple of an odd prime. Now we will consider the case where n does not have an odd prime factor, namely $n = 2^k$, since we have eliminated all values except for odd primes and the numbers $4, 8, 16, 32, 64, ..., 2^k$. Using the same logic, we will prove that Fermat's Last Theorem holds for exponents of the form 2^k for $k \geq 2$ if it holds for exponent 4.

Theorem 3. If equation (1) has a solution for $n = 2^k$, for $k \geq 2$, then it has a solution for $n = 4$.

Proof. Assume (x, y, z) is a solution for $n = 2^k$. So, $x^{2^k} + y^{2^k} = z^{2^k}$ and $2^k = 4 * 2^{k-2}$. Substituting, we see $x^{(2^{k-2})4} + y^{(2^{k-2})4} = z^{(2^{k-2})4}$. Thus, $(x^{2^{k-2}}, y^{2^{k-2}}, z^{2^{k-2}})$ is a positive integer solution for $n = 4$. $\square$

Since we have mentioned already that Fermat proved Fermat's Last Theorem holds for $n = 4$, the previous theorem shows that Fermat's Last Theorem for any exponent of the form 2^k. Based on the proofs above, we only need to establish Fermat's Last Theorem for odd prime integers. Prior to Germain, most mathematicians who worked on Fermat's Last Theorem tried to prove it for specific values of n. Germain was the first to attempt to prove Fermat's theorem for an infinite number of exponents and successfully worked towards a general proof for many primes, though not all.

8 The Sophie Germain Theorem

In her letters to Gauss, Sophie Germain discusses her plan to prove Fermat's Last Theorem in its entirety. As a result of her work, Fermat's Last Theorem was eventually split into two cases:

- Case 1: $x^p + y^p = z^p$ has no solution when p does not divide xyz
- Case 2: $x^p + y^p = z^p$ has no solution when exponent p does divide xyz [7].

She believed that if for a given prime exponent p there exist infinitely many auxiliary primes such that its nonzero p^{th} power residues are nonconsecutive, then Case 1 of Fermat's Last Theorem holds for that exponent [10].

We will see that a similar condition is used in her attempted proof of Case 1 of Fermat's Last Theorem; the only difference being that her proof requires only a single auxiliary prime. Germain noted that because each auxiliary prime must divide either x, y, or z, then "the existence of infinitely many of them will make Fermat's equation impossible"[10]. Germain worked relentlessly towards the completion of this proof and made great progress with her method, but unfortunately, she was unable to fully develop it. She did, however, claim "that any solutions to a Fermat equation would have to 'frighten the imagination' with their size" [10].

Her methods were mainly focused on proving the Condition N-C (Non-Consecutivity) which states, "*There do not exist two nonzero consecutive p^{th} power residues, modulo θ*" for infinitely many auxiliary primes of the form $\theta = 2Np + 1$ [10]. A p^{th} power residue modulo θ can be found by raising integers up to the auxiliary prime, θ, to the power of the prime, p, and then taking each of these results modulo θ. The resulting residues should not be consecutive powers. Observe the example below where $p = 3$.

To determine if $p = 3$ satisfies the Condition N-C we will compute x^3 mod 7. We consider modulo 7, because 7 is the auxiliary prime, such that $2Np + 1 = 7$ which was obtained from $2 * 1 * 3 = 7$ and 7 is prime.

1^3	1	1 mod 7	1
2^3	8	8 mod 7	1
3^3	27	27 mod 7	6
4^3	64	64 mod 7	1
5^3	125	125 mod 7	6
6^3	216	216 mod 7	6

To solve 1 mod 7, one divides 1 by 7, noting that it goes in zero times with a remainder of 1. The remainder is the answer and the residue of 1^3 mod 7. Likewise, for 27 mod 7, we divide 27 by 7 and see that 7 goes into 27 three times with a remainder of 6, thus we have a residue of 6. The only residues that result from these computations are 1 and 6, which are nonconsecutive, as 0 is a potential residue power as well. These residues can also be referred to as ±1 as they will be referenced in the next table. The residue 6 is equivalent to a residue of -1.

Germain makes an additional claim that the number 2 is not a p^{th} power residue. For the example above when $p = 3$, we can observe that this claim also holds. Below is the table of prime numbers, p less than 100 for which Sophie Germain computed the same calculations as shown above. It also includes the auxiliary prime q and the residues for each value [13]. As the numbers grew in size, so did the extent of the computations.

p	$q = 2Np + 1$	R
3	$2 * 1 * 3 + 1 = 7$	± 1
5	$2 * 1 * 5 = 11$	± 1
7	$2 * 2 * 7 + 1 = 29$	$\pm 1, \pm 12$
11	$2 * 1 * 11 + 1 = 23$	± 1
13	$2 * 2 * 13 + 1 = 53$	$\pm 1, \pm 23$
17	$2 * 4 * 17 + 1 = 137$	$\pm 1, \pm 10, \pm 37, \pm 41$
19	$2 * 5 * 19 + 1 = 191$	$\pm 1, \pm 7, \pm 39, \pm 49, \pm 82$
23	$2 * 1 * 23 + 1 = 47$	± 1
29	$2 * 1 * 29 + 1 = 59$	± 1
31	$2 * 5 * 31 + 1 = 311$	$\pm 1, \pm 6, \pm 36, \pm 52, \pm 95$
37	$2 * 2 * 37 + 1 = 149$	$\pm 1, \pm 44$
41	$2 * 1 * 41 + 1 = 83$	± 1
43	$2 * 2 * 43 + 1 = 173$	$\pm 1, \pm 80$
47	$2 * 7 * 47 + 1 = 659$	$\pm 1, \pm 12, \pm 55, \pm 144, \pm 249, \pm 270, \pm 307$
53	$2 * 1 * 53 + 1 = 107$	± 1
59	$2 * 7 * 59 + 1 = 659$	$\pm 1, \pm 20, \pm 124, \pm 270, \pm 337, \pm 389, \pm 400$
61	$2 * 8 * 61 + 1 = 977$	$\pm 1, \pm 52, \pm 80, \pm 227, \pm 252, \pm 357, \pm 403, \pm 439$
67	$2 * 2 * 67 + 1 = 269$	$\pm 1, \pm 82$
71	$2 * 4 * 71 + 1 = 569$	$\pm 1, \pm 76, \pm 86, \pm 277$
73	$2 * 2 * 73 + 1 = 293$	$\pm 1, \pm 138$
79	$2 * 2 * 79 + 1 = 317$	$\pm 1, \pm 114$
83	$2 * 1 * 83 + 1 = 167$	± 1
89	$2 * 1 * 89 + 1 = 179$	± 1
97	$2 * 2 * 97 + 1 = 389$	$\pm 1, \pm 115$

For each prime less than 100, both of Germain's conditions were satisfied, such that there are no consecutive p^{th} power residues and none of the residues are the number 2.

The form $\theta = 2Np + 1$ came from Germain's initial work on what are known today as Sophie Germain primes. A Sophie Germain prime is a prime, p, such that $2p + 1$ is also prime. The primes below 100 for which this is satisfied are listed below.

```
for p to 100 do
    if (isprime(p) then
        if isprime(2*p+1) then
            print(p)
        end if
    end if
end do;
2, 3, 5, 11, 23, 29, 41, 53, 83, 89
```

Because there are additional primes that are not Sophie Germain primes less than 100, Legendre extended the criterion such that p is prime and $4p + 1$, or $8p + 1$, or $10p + 1$, or $14p + 1$, or $16p + 1$ is prime in order to encompass all of the prime values less than 100 [13]. Germain, with the help of Legendre, thus modified her plan to require the auxiliary prime to satisfy the general case such that p is prime and $2Np + 1$ is also prime, now encompassing all primes less than 100.

```
for i from 197 to 500 do
    if (isprime(i)) then
        if (isprime(2*i+1) or isprime(4*i+1) or isprime(8*i+1) or isprime(10*i+1) or
        isprime(14*i+1) or isprime(16*i+1) then
            print(i)
        end if
    end if
end do;
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
```

Germain outlined a plan to prove other numbers had the requisite form following a method of induction. Though her method of induction did not come completely into fruition, “her instincts here were correct, as [was eventually] proven by Wendt” in 1894 [13]. While she did not actually prove the

Sophie Germain Theorem, mathematicians named the following proof acknowledging the contributions of Sophie Germain

The Sophie Germain Theorem. If p is prime and $2Np + 1$ is prime and there are no consecutive p^{th} power residues such that p does not divide xyz, then there are no solutions for Case 1 of Fermat's Last Theorem.

Sophie Germain's great contribution, for which she became famous, are the ideas that led to a proof of the first case of Fermat's Last Theorem. "It is not surprising that she wishes to claim to have proven case 1 of Fermat's Last Theorem, even though she still recognizes that there are implicit hypotheses she has not completely verified for all exponents" [13]. For the first case of Fermat's Last Theorem, Germain claims to eliminate solutions so that none are divisible by p, in other words, $p \nmid xyz$ where p is prime and has an auxiliary prime $2Np + 1$ such that there are no consecutive p^{th} power residues and p is not a p^{th} power residue. Sophie Germain's ideas led to the first great stride made in solving Fermat's Last Theorem. She laid the foundation for a number of mathematicians in the future to continue making progress in the field of number theory and specifically on Fermat's Last Theorem. Sophie Germain and her work have left a significant impact on the field of mathematics as it is still being studied today.

9 Extensions of The Sophie Germain Theorem Today

Sophie Germain's work on Fermat's Last Theorem made significant progress for her time, but it left many people curious, wanting to discover more. Many people continued to work with her ideas as they were driven to prove Fermat's Last Theorem for odd primes greater than 100 and eventually all numbers. Though her goal was a proof by mathematical induction, Germain made numerous time-consuming calculations for each individual number in her efforts to prove the theorem. Others continued to search for a different method to prove Fermat's Last Theorem in its entirety.

As we mentioned earlier, Adrien-Marie Legendre extended Germain's work by adding a modification to her original idea. His contribution was the idea that the auxiliary prime need not be $2p + 1$, but could also be $q = 4p + 1$, or $q = 8p + 1$, or $q = 10p + 1$, or $q = 14p + 1$, or $q = 16p + 1$ in order to capture more prime numbers. With this addition to Germain's idea, Legendre and Germain successfully completed the computations for odd primes less than 197. This is where his pattern first failed. Like Germain, due to the constraints of technology at the time, he also computed these by hand. To our benefit, technology today allows us to see the extent of his contribution to primes greater than 197. The Maple code below produces a list of primes between 197 and 500 that his extension also accounted for. You can imagine that this pattern extends for many primes larger than 500 as well.

```
for i from 197 to 500 do
    if (isprime(i)) then
        if (isprime(2*i+1) or isprime(4*i+1) or isprime(8*i+1) or isprime(10*i+1) or
        isprime(14*i+1) or isprime(16*i+1) then
            print(i)
        end if
    end if
end do;
199, 211, 233, 239, 241, 251, 269, 271, 277, 281, 293, 307, 313, 331, 337, 347, 349, 353, 359, 367,
373, 397, 401, 409, 419, 421, 431, 433, 439, 443, 449, 479, 487,
    491, 499
```

Furthermore, we might be curious as to why Legendre chose $q = ap + 1$ with a being 4, 8, 10, 14, and 16. He continued her pattern with multiples of 2 which are not divisible by 3. With Legendre's theorem, we find that we are still missing some odd prime numbers. Those less than 500 have been obtained using Maple and are seen below.

```
for i from 1 to 500 do
    if isprime(i) then
        if 'not' (isprime(2*i+1) or isprime(4*i+1) or isprime(8*i+1) or isprime(10*i+1) or
        isprime(14*i+1) or isprime(16*i+1)) then
            print(i)
        end if
    end if
```

```
end do;
197, 223, 227, 229, 257, 263, 283, 311, 317, 379, 383, 389, 457, 461, 463, 467
```

Again, with the use of our current technology, we can easily extend Legendre's theorem to include numbers where p is prime and $q = ap + 1$ is prime for $a = 20$, $a = 22$, $a = 26$, $a = 28$, $a = 32$, and all even numbers that do not have a factor of 3. If we remove primes where q is prime for the values mentioned above, we have now reduced the pool of remaining numbers to be proven for Fermat's Last Theorem under 500 to the eight below.

```
197, 223, 257, 283, 383, 389, 457, 463
```

Using Maple, we can quickly solve for the value of a that will suit these remaining numbers. It is easy to see why Legendre stopped at $a = 16$, for the calculations required for larger values of a would not have been realistic for the time. We could do this for every number if we wanted and continue the pattern of Legendre's theorem to completely satisfy the first case of Fermat's Last Theorem. It became known that the auxiliary prime could statisfy any $2Np + 1$. This, however, was not the method utilized to continue Germain's original proof due to computational technology having yet to be discovered. Despite Legendre's efforts, the proof was also not one by induction and thus would require us to follow this method for infinitely many primes.

In the aftermath of Sophie Germain and Adrien-Marie Legendre's breakthrough, a prize was offered to the mathematician who could prove of Fermat's Last Theorem in its entirety. Gabriel Lamé, Augustin Cauchy, and Ernst Kummer were among the notable mathematicians who worked to solve the long-time mystery. Putting the problem to rest for another century, "Kummer had demonstrated that a complete proof of Fermat's Last Theorem was beyond the current mathematical approaches" [15]. There were others who also began to suggest its impossibility. This was devastating to the mathematical community who had high hopes of solving "the world's hardest mathematical problem" [15].

In his youth, Andrew Wiles studied the works of Germain and others who also worked on Fermat's Last Theorem, including Kummer who doubted the current methods to prove the theorem. Ken Ribet, Gerhard Frey, and Barry Mazur worked to discover that Fermat's Theorem was closely related to the Taniyama-Shimura conjecture. "If somebody could prove that every elliptic equation is modular, then this would imply that Fermat's equation had no solutions, and immediately prove Fermat's Last Theorem" [15]. Still, this did not make the problem any easier as the Taniyama-Shimura conjecture had gone unproven for 30 years. Again, many were doubtful that anyone could prove this theorem. Ribet claimed,

> "I was one of the vast majority of people who believed that the Taniyama-Shimura conjecture was completely inaccessible. I didn't bother to try and prove it. I didn't even think about trying to prove it. Andrew Wiles was probably one of the few people on earth who had the audacity to dream that you can actually go and prove this conjecture" [15].

Despite the pessimism displayed by Ribet, Andrew Wiles indeed proved the Taniyama-Shimura conjecture. After years of intense work, he first published his proof in 1993, but after discovering a mistake in his proof, he worked on it again for another year, officially proving the Taniyama-Shimura conjecture in 1994. Over 350 years later, Fermat's Last Theorem was finally proven. Still today many continue to study Fermat's Last Theorem. Though there exists a proof, a proof of Fermat's Last Theorem using number theory would lead to immense strides and greater understanding in the field of number theory as a whole.

10 The Legacy of Sophie Germain

Figure 4: Sophie Germain's tombstone at Père Lachaise Cemetary in Paris.

Sophie Germain died on June 27, 1830 from breast cancer. Prior to her death, Carl Freidrich Gauss recommended her for an honorary degree from the University of Göttingen, but she died before she had the chance to receive it. Even after her death, it took time for Germain to receive the recognition that she deserved. When completing her death certificate, the state official refused to indicate her profession as *mathematician* and instead only assigned her the title of *property owner* [17]. The Eiffel Tower has seventy-two names engraved to honor those who made contributions to the mathematics of elasticity so that the construction of the Eiffel Tower would be possible [18]. Germain, though, despite her work in elasticity, was not one of the seventy-two. There are still some instances in which Germain has not received the recognition that she deserves, but appreciation for her work as a woman eventually grew and has been represented in various settings within Paris.

Figure 5: Rue Sophie Germain in Paris.

Sophie Germain's name now appears as a street name in Mathematical Paris, an area in Paris where many of the street are named after renowned mathematicians. She also has a school named after her in Paris; in the courtyard at École Sophie Germain, there stands a statue in her honor. The Sophie Germain Hotel is also named after her, and her home is now considered an historical landmark in France [14].

Figure 6: This plaque is outside of the home in which Sophie Germain lived.

Sophie Germain has also been honored on a stamp that was recently made available in France March 18, 2016 with a simple image of her mathematical conclusion regarding Fermat's Last Theorem, some of her mathematical drawings, and a sketch of Germain, herself.

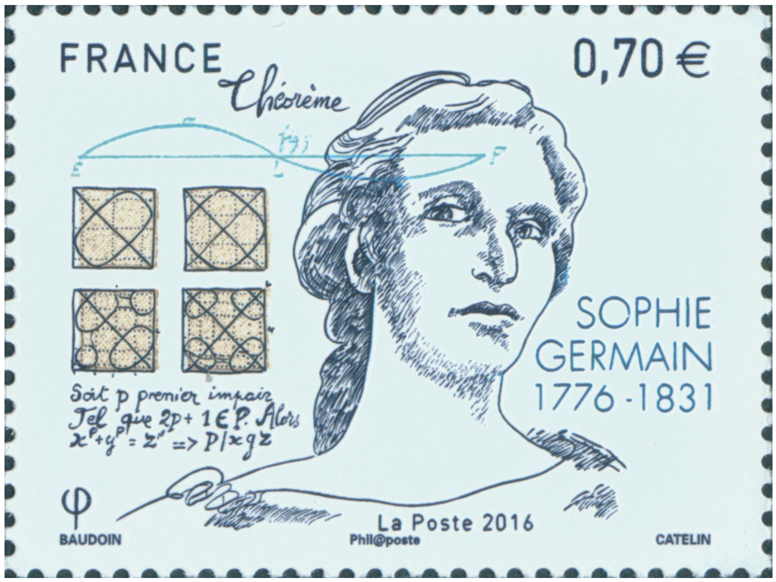


Figure 7: The stamp featuring Sophie Germain circa 2016.

Sophie Germain has made an impact on many women in academia as a result of her motivation and perseverance to pursue an education and make discoveries in mathematics. In a personal interview with French female mathematician, Amy Dahan, who works at the French national Centre for Scientific Research, said, "Au début, je n'ai pas travaillé sur Sophie Germain parce qu'elle était une femme mais parce qu'elle s'était intéressée aux théories de l'élasticité une branche de la physique mathématique de l'époque qui recoupait mon sujet de thèse. Ensuite, j'ai été frappée par sa trajectoire personnelle, et j'ai écrit [quelques] papiers sur elle" [3]. (At first, I did not work on Sophie Germain because she was a woman, but because she was interested in theories of elasticity, a branch of mathematical physics which, at the time, intersected the subject of my thesis. Then, I had been struck by her personal trajectory, and I wrote some papers on her.) Although her original research was not primarily focused on Sophie Germain, rather her work in elasticity, Dahan, like many other young women pursuing a career in a predominately male-dominated field, she found herself intrigued and impressed by Sophie Germain's story.

11 Conclusion

Sophie Germain, born in 1776 and who grew up as a teenager in Paris during the time of the French Revolution and the Enlightenment, faced many challenges in obtaining an education. During a time that women were deprived of receiving a formal education, Germain used any methods available to her to self-educate herself. She started studying mathematics and science as a young girl, reading many of the books found in her father's library. Germain lacked the support of her parents for much of her youth, as they took away all sources of heat and light in an unsuccessful attempt to deter her from her love of mathematics. It was uncommon for a woman to study subjects outside of religion and the arts at the time, causing her parents did to disapprove of their daughter's strange interest.

As a result of the educational inequalities during the time of the French Revolution, women were only able to share their academic ideas if they were related to another scientist, had a personal connection with a scientist, or attended salons; otherwise, there was little to no platforms available to women to share their notes in the field of academia. Sophie Germain, upon the opening of the École Polytechnique, was able to obtain the lecture notes of Adrien-Marie Legendre and Joseph Louis Lagrange. She, later on, wrote to both of these professors under a male student's name, Monsieur LeBlanc. Through her communication with Legendre and Carl Freidrich Gauss, Germain gained much knowledge and was better able to penetrate the field of mathematics, despite her gender. Both were pleased and impressed to learn of her true identity and offered her much support in her endeavors. Her father also became more supportive over time, aiding her financially and allowing scientists to come visit their home to work with Germain.

Sophie Germain worked in elasticity and number theory, with her main contributions being her work on Fermat's Last Theorem. Germain's work led to a proof of the first case of Fermat's Last Theorem for p is prime, $2Np + 1$ is an auxiliary prime, and $p \nmid xyz$. She did the computations for $p < 100$ with the intention to provide a proof by induction for all primes. Though she did not completely prove Fermat's Last Theorem, she provided a sound base for future mathematicians. It was not until 1994 that Andrew Wiles officially proved Fermat's Last Theorem for all exponents. He did so by proving the Taniyama-Shimura conjecture, a different approach than Germain. There are a number of mathematicians today who are still studying Fermat's Last Thereom and searching for a proof using number theory.

Sophie Germain paved the way for future women mathematicians both in France and throughout the world. She overcame a great deal of adversity, making it clear that a woman could be influential in any sphere that she wishes. Though she was not honored immediately by many, today her contributions to mathematics are greatly acknowledged. She is honored in Paris with schools, a road, and a hotel named after her. Her home is considered an historical landmark, and she was featured on a postage stamp recently in 2016. Sophie Germain will continue to be honored for her great work and her legacy will live on as mathematics will never die, rather it will continue to grow, be strengthened, and improved upon.

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